4. Y. Mori and W. Nakayama, Study on forced convective heat transfer in curved pipes (3rd report, theoretical analysis under the condition of uniform wall temperature and practical formulae), *Int. J. Heat Mass Transfer* **10**, 681–695 (1967).
5. A. N. Dravid, K. A. Smith, E. W. Merrill and P. L. T. Brian, Effect of secondary fluid motion on laminar flow heat transfer in helically coiled tubes, *A.I.Ch.E. Jl* **17**, 1114–1122 (1971).
6. M. N. Özisik and A. C. Topakoglu, Heat transfer for laminar flow in a curved pipe, *J. Heat Transfer* **90**, 313–318 (1968).
7. K. C. Cheng, G. J. Hwang and M. Akiyama, On a simple correlation for Prandtl number effect on forced convective heat transfer with secondary flow, *Int. J. Heat Mass Transfer* **15**, 172–175 (1972).
8. H. Ito, Friction factors for turbulent flow in curved pipes, *J. Basic Engng* **81D**, 123–134 (1959).
9. J. A. Baylis, Experiments on laminar flow in curved channels of square section, *J. Fluid Mech.* **48**, 417–422 (1971).

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DROPWISE CONDENSATION OF MERCURY

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NOMENCLATURE

$A(r)\mathrm{d}r$,	fractional area covered by drops having base radius in the range $r, r + \mathrm{d}r$;
f,	condensation coefficient;
g,	gravitational acceleration;
h_{fg},	specific enthalpy of phase change;
K_1,	a shape factor, see equations (20) and (21);
K_2^*,	ratio of base area to curved surface area of a drop;
K_2,	defined in equation (22);
K_3,	a constant defined in equation (10);
L_p,	thickness of promoter layer;
L_0,	$2\sigma v_f/h_{fg}$
L_0^*,	$L_0 \sin\beta$;
L_3,	$\sqrt{[\sigma/g(\rho_f - \rho_g)]}$;
$\dot{m}''$,	net mass flux across vapour–liquid interface;
n,	a constant in the drop-size distribution, see equation (15);
$\dot{Q}''$,	average heat flux for condensing surface;
$\dot{Q}_b''$,	heat flux through base area of a drop;
$\dot{Q}_1''$,	$\lambda_f T h_{fg}/2\sigma v_f$;
$\dot{Q}_1''^*$,	$\dot{Q}_1''/\sin\beta$;
$\dot{Q}_2''$,	$h_{fg}^2/\phi v_g\sqrt{(RT)}$;
q,	non-dimensionalized heat flux defined in equation (17);
R,	specific ideal-gas constant of vapour;
r,	base radius of a drop;
$\check{r}$,	minimum base radius of a drop;
$\hat{r}$,	maximum base radius of a drop;
r_c,	radius of curvature of curved surface of a drop;
T,	thermodynamic temperature;
v_f,	specific volume of condensate;
v_g,	specific volume of vapour;
α,	vapour-to-surface heat-transfer coefficient;
β,	contact angle;
γ,	ratio of isobaric to isochoric specific heat capacity;
ΔP,	interface pressure difference due to interphase matter transfer;
ΔT,	effective vapour-to-surface temperature difference;
ΔT_c,	$2\sigma v_f T/r_c h_{fg}$;
ΔT_d,	effective temperature difference across a drop due to conduction in the drop, see equation (20);
θ,	non-dimensionalized temperature difference defined in equation (18);
θ_0,	defined in equation (19);
λ_f,	thermal conductivity of condensate;
λ_p,	thermal conductivity of promoter layer;
ρ_f,	density of condensate;
σ,	liquid–vapour interfacial tension;
ϕ,	$\Delta P/\dot{m}''\sqrt{(RT)}$.

1. INTRODUCTION

A recent theory of dropwise condensation heat transfer [1] has received strong support from a variety of experimental investigations [2–10] for dropwise condensation of steam over wide ranges of heat flux and pressure. In the theory [1], drops were considered to be hemispheres (a satisfactory assumption in the case of water) and thus no distinction was made between the radius of the base of a drop and the radius of curvature of the curved surface. In the present work the theory is modified to cover cases where drops do not approximate to hemispheres and compared with recent measurements for dropwise condensation of mercury [11].

The radius of curvature of the liquid–vapour interface plays a role through its effect on the relation between the equilibrium temperature and the pressures on either side of the interface. This gives rise to an effective resistance to heat transfer which is important for very small drops, and also determines the size of the smallest viable drop. The base radius of a drop is used, together with expressions for the heat-transfer rate through a drop of given size and the distribution of drop sizes, to calculate the average heat flux for the condensing surface.

2. THE MODIFIED THEORY

With the assumption adopted earlier [1], the amount ΔT_c by which the equilibrium temperature for a curved interface is less, for the case of the liquid drop, then the equilibrium ("saturation") temperature for a plane interface for the same vapour pressure is given by

$$\Delta T_c = \frac{2\sigma v_f T}{r_c h_{fg}} \tag{1}$$

where r_c is the radius of curvature of the liquid–vapour interface. When drops are treated as segments of spheres

$$r = r_c \sin\beta \tag{2}$$

where r is the radius of the base of a drop and β is the contact angle. Thus, for a vapour-to-surface temperature difference ΔT during dropwise condensation, the base radius of the smallest drop which can exist is given by

$$\check{r} = \frac{2\sigma v_f}{h_{fg}} \frac{T}{\Delta T} \sin\beta. \tag{3}$$

When account is taken of the effects of surface curvature, interphase matter transfer, conduction in the drop and in the promoter layer beneath it as before [1], the heat flux, $\dot{Q}_b''$, through the base of a drop of base radius r is given by

$$\dot{Q}_b'' = \frac{\Delta T - \dfrac{2\sigma v_f T \sin\beta}{r h_{fg}}}{\dfrac{K_1 r}{\lambda_f} + \dfrac{K_2 \phi v_g T \sqrt{(RT)}}{h_{fg}^2}}. \tag{4}$$

For hemispherical drops, when $\beta = \pi/2$, equations (3) and (4) give the expressions used earlier [1] for $\check{r}$ and $\dot{Q}_b''$.

The equation for the interphase pressure drop used in (1)

$$\Delta P = \dot{m}'' \frac{(\gamma + 1)}{(\gamma - 1)} \sqrt{(RT/2\pi)} \tag{5}$$

gives

$$\phi = (\gamma + 1)/(\gamma - 1)\sqrt{(2\pi)}. \tag{6}$$

From equation (6) values of ϕ of about 1·6 for mercury and about 2·8 for steam are obtained. Since there is as yet no *established* theory of interphase matter transfer it is of interest to compare the above values of ϕ with that obtained from an alternative commonly used (see for instance [12]) expression for ΔP

$$\Delta P = \dot{m}'' \left(\frac{2 - f}{2f}\right) \sqrt{(2\pi RT)} \tag{7}$$

where f is the so-called condensation or evaporation coefficient. Equation (7) gives

$$\phi = \frac{2 - f}{2f} \sqrt{(2\pi)}. \tag{8}$$

It seems now widely to be thought that $f = 1$, giving a value of ϕ of about 1·25. Since the theory of dropwise condensation includes constants whose values are not precisely known, it is clear that, by using only slightly modified values of the constants, agreement between theory and measurements would be equally satisfactory when using equation (7) to provide a value of ϕ, rather than equation (5) as in [1].

The mean heat flux for the surface as a whole is given by

$$\dot{Q}'' = \pi\Sigma(\dot{Q}_b'' r^2) \tag{9}$$

where the summation is taken over all drops on unit area of the surface, from the smallest having base radius $\check{r}$ to the largest having base radius $\hat{r}$.

As before [1] we estimate

$$\hat{r} = K_3[\sigma/g(\rho_f - \rho_g)]^{\frac{1}{2}} \tag{10}$$

where K_3 is a constant of order of magnitude unity, and $\check{r}$ is given by equation (3). Thus, when using a continuous function for the distribution of drop sizes, the average heat flux for the whole surface is given by

$$\dot{Q}'' = \int_{\check{r}}^{\hat{r}} \dot{Q}_b'' A(r) \mathrm{d}r \tag{11}$$

where $A(r)\,\mathrm{d}r$ is the fractional area of the condenser surface covered by drops having radii in the interval r, $r + \mathrm{d}r$.

From equations (3), (4) and (11)

$$\dot{Q}'' = \int_{\frac{TL_0 \sin\beta}{\Delta T}}^{\hat{r}} \left\{ \frac{\dfrac{\Delta T}{T} - \dfrac{L_0 \sin\beta}{r}}{\dfrac{K_1 r}{\dot{Q}_1'' L_0} + \dfrac{K_2}{\dot{Q}_2''}} \right\} A(r)\,\mathrm{d}r \tag{12}$$

where $\dot{Q}_1''$, $\dot{Q}_2''$ and L_0 are properties of water substance (see Nomenclature). Equation (12) differs from that used earlier

[1] in that integrand and lower limit involve sin β. However, if we define new quantities L_0^* and $\dot{Q}_1''^*$

$$L_0^* = L_0 \sin\beta \tag{13}$$

$$\dot{Q}_1''^* = \dot{Q}_1''/\sin\beta \tag{14}$$

and substitute for L_0 and $\dot{Q}_1''$ in equation (12), the latter becomes identical in form (with L_0^* and $\dot{Q}_1''^*$ replacing L_0 and $\dot{Q}_1''$) to the equation for $\dot{Q}''$ used originally [1].

With the distribution function used earlier [1] for the distribution of drop sizes

$$A(r) = \frac{1}{n\hat{r}}\left(\frac{r}{\hat{r}}\right)^{[(1/n)-1]} \tag{15}$$

where n is a constant of order of magnitude unity. The integral in equation (12) may be evaluated as before [1] and the result expressed in non-dimensional form

$$q = \theta_0^{1/n} F_n(\theta) \tag{16}$$

$$q = \left(\frac{K_2^2}{K_1}\frac{\dot{Q}_1''^*}{\dot{Q}_2''^2}\right)\dot{Q}'' = \left(\frac{K_2^2}{K_1}\frac{\dot{Q}_1''}{\dot{Q}_2''^2\sin\beta}\right)\dot{Q}'' \tag{17}$$

$$\theta = \left(\frac{K_2}{K_1}\frac{\dot{Q}_1''^*}{\dot{Q}_2''}\frac{1}{T}\right)\Delta T = \left(\frac{K_2}{K_1}\frac{\dot{Q}_1''}{\dot{Q}_2''\sin\beta}\frac{1}{T}\right)\Delta T \tag{18}$$

$$\theta_0 = \frac{K_2}{K_1K_3}\frac{L_0^*}{L_3}\frac{\dot{Q}_1''^*}{\dot{Q}_2''} = \frac{K_2}{K_1K_3}\frac{L_0}{L_3}\frac{\dot{Q}_1''}{\dot{Q}_2''}. \tag{19}$$

Expressions for F_n for different values of n are given in [1].

3. VALUES OF THE CONSTANTS AND COMPARISON WITH MEASUREMENTS

Each of the four constants K_1, K_2, K_3 and n should have a value close to unity but none is known precisely. Further, the appropriate values for mercury might be expected to differ from those for water.

The "effective" temperature difference across a drop ΔT_d due to conduction in the drop is estimated [1] as

$$\Delta T_d = K_1 \frac{r}{\lambda_l}\dot{Q}_b'' \tag{20}$$

which defines K_1. If we model a drop as a cylinder standing on the same base as the drop and having the same volume and adiabatic curved surface then

$$K_1 = \frac{2 - 3\cos\beta + \cos^3\beta}{3\sin^3\beta}. \tag{21}$$

Thus for hemispherical drops $K_1 = \frac{2}{3}$ (the value adopted earlier [1] for steam) and for $\beta = 143°$ (for dropwise condensation of mercury on stainless steel [11]) $K_1 \simeq 6$. It is not of course suggested that such a model gives anything other than a crude estimate of K_1.

The constant K_2 arises from a combination of the terms for interphase temperature drop and the temperature drop in the promoter layer, and is given by

$$K_2 = K_2^* + L_p\dot{Q}_2''/\lambda_p T \tag{22}$$

where K_2^* is the ratio of base area to area of the curved surface of a drop.

For the case of mercury, where there is no promoter layer

$$K_2 = K_2^* = (1 + \cos\beta)/2 \tag{23}$$

giving, with $\beta = 143°$,

$$K_2 = K_2^* \simeq 0.1.$$

It is more difficult to provide even crude estimates for values of K_3, the constant related to $\hat{r}$ (see equation (10)) and n the constant in the drop size distribution (see equation (15)). The drop size distribution is affected both by growth rate of individual drops and by coalescences between drops. For contact angles exceeding $\pi/2$ coalescence does not take place when the base circles of neighbouring drops touch as in the case of hemispherical and smaller drops. As pointed out earlier [13] and more recently in greater detail [14], $\hat{r}$ is an "average" maximum drop radius and not simply the radius of the largest drop which can remain on the surface. Hence K_3 is not simply proportional to the base radius of the largest drop. A further complication which could influence any attempt to determine both n and K_3 for mercury is the report [15] that during dropwise condensation of mercury drops become detached from the surface and do not sweep vertical tracks as in the case of water. Thus, apart from the fact that n and K_3 should not differ widely from unity, it is not possible, at this stage, to obtain more precise estimates.

Ivanovskii *et al.* [11] found that for dropwise condensation of mercury on a polished stainless steel surface, the vapour-to-surface heat-transfer coefficient increased with time for several days until a steady value was reached. This increase was attributed to the removal of oxide. For a vertical surface they observed that the steady maximum coefficient was essentially independent of vapour-to-surface temperature difference except for temperature differences below about 2 K when a fall in the coefficient is reported. Figure 1 shows the measured dependence of the coefficient (presumably for vapour-to-surface temperature differences exceeding 2 K) on the vapour temperature for a vertical condensing surface.

The lines shown on Fig. 1 are those given by the modified theory, with $n = 3$, $K_1 = 6$, $K_2 = K_2^* = 0{\cdot}1$ and $K_3 = 4{\cdot}28$, for two values of the vapour-to-surface temperature difference judged to represent the approximate limits of the experimental values. It was not possible to determine the "best" values of the constants since the experimental heat fluxes and temperature differences are not given. By considering that each individual point represented two experiments, one with $\Delta T = 4$ K and another with $\Delta T = 12$ K, and adopting the above values for K_1 and K_2, minimization of the sum of the squares of either absolute or relative residuals of $\dot{Q}''$ indicated values of n and K_3 close to those given above. For convenience, since this value was used earlier for water, $n = 3$ was adopted and the value of 4·28 for K_3 was obtained by minimization of the sum of the squares of residuals of ln (α).

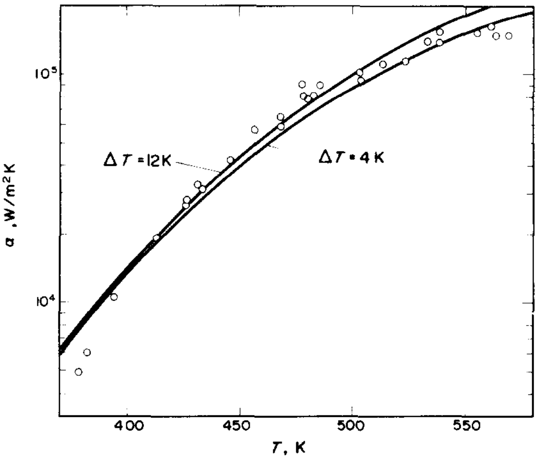

FIG. 1. Comparison between theory and measurements [11].

4. CONCLUDING REMARKS

It may be seen from Fig. 1 that the theory indicates a small increase in α with ΔT where the experiments showed such an increase only for ΔT less than about 2 K. Also, the curvature of the theoretical lines is not so great as that exhibited by the experimental points. However, it is clear from Fig. 1 that the theory and observations are in fair agreement over the whole range of T and ΔT when the constants do not differ widely from their expected values. In view of the wide differences in properties, notably thermal conductivity, between water and mercury, the theory receives strong support from the fact that it is found to be generally satisfactory in both cases.

REFERENCES

1. E. J. LE FEVRE and J. W. ROSE, A thoery of heat transfer by dropwise condensation, Proc. Third Int. Heat Transfer Conference, Am. Instn. Chem. Engrs, vol. 2, p. 362 (1966).
2. H. WENZEL, Versuche über Tropfenkondensation, *Allg. Wärmetech.* **8**, 53 (1957).
3. E. J. LE FEVRE and J. W. ROSE, Heat-transfer measurements during dropwise condensation of steam, *Int. J. Heat Mass Transfer* **7**, 272 (1964).
4. D. W. TANNER, C. J. POTTER, D. POPE and D. WEST, Heat transfer in dropwise condensation—Part I, *Int. J. Heat Mass Transfer* **8**, 419 (1965).
5. D. W. TANNER, D. POPE, C. J. POTTER and D. WEST, Heat transfer in dropwise condensation—Part II, *Int. J. Heat Mass Transfer* **8**, 427 (1965).
6. E. J. LE FEVRE and J. W. ROSE, An experimental study of heat-transfer by dropwise condensation, *Int. J. Heat Mass Transfer* **8**, 1117 (1965).
7. D. W. TANNER, D. POPE, C. J. POTTER and D. WEST, Heat transfer in dropwise condensation at low steam pressures in the absence and presence of non-condensable gas, *Int. J. Heat Mass Transfer* **11**, 181 (1968).
8. E. CITAKOGLU and J. W. ROSE, Dropwise condensation—some factors influencing the validity of heat-transfer measurements, *Int. J. Heat Mass Transfer* **11**, 523 (1968).
9. C. GRAHAM, The limiting heat transfer mechanisms of dropwise condensation, Ph.D. Thesis, Mass. Inst. Tech. (1969).
10. R. WILMSHURST and J. W. ROSE, Dropwise condensation—further heat-transfer measurements. Proc. Fourth Int. Heat Transfer Conference 6, paper Cs 1.4 (1970).
11. M. N. IVANOVSKII, V. I. SUBBOTIN and YU. V. MILOVANOV, Heat transfer with dropwise condensation of mercury vapour, *Teploenergetika* **14** (6), 81 (1967).
12. S. J. WILCOX and W. M. ROHSENOW, Film condensation of potassium using copper condensing block for precise wall-temperature measurement, *J. Heat Transfer* **92C**, 359 (1970).
13. E. J. LE FEVRE and J. W. ROSE, Dropwise condensation, Proceedings of the symposium to celebrate the bicentenary of the James Watt Patent, Univ. Glasgow, 166 (1969).
14. J. W. ROSE and L. R. GLICKSMAN, Dropwise condensation—the drop size distribution, submitted for publication 1972.
15. M. KOLLERA and U. GRIGULL, Über das Abspringen von Tropfen bei der Kondensation von Quecksilber, *Wärme-und Stoffübertragung* **2**, 31 (1969).

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THE RESPONSE OF HEATED WIRE TEMPERATURE DISTRIBUTIONS TO UNSTEADY SURFACE COOLING RATES

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